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DNS OF MULTIPLICITY AND STABILITY OF MIXED CONVECTION IN ROTATING CURVED DUCTS

LIQIU WANG AND TIANLIANG YANG
Department of Mechanical Engineering
The University of Hong Kong, Hong Kong

Abstract. A numerical study is made on the fully-developed bifurcation structure and stability of the mixed convection in rotating curved ducts with the emphasis on the effect of buoyancy force. The rotation can be positive or negative. The fluid can be heated or cooled. The study reveals the rich solution and flow structures and complicated stability features. One symmetric and two symmetric/asymmetric solution branches are found with seventy-five limit points and fourteen bifurcation points. The flows on these branches can be symmetric, asymmetric, 2-cell and up to 14-cell structures. Dynamic responses of the multiple solutions to finite random disturbances are examined by the direct transient computation. It is found that possible physically realizable fully-developed flows evolve, as the variation of buoyancy force, from a stable steady multi-cell state at a large buoyancy force of cooling to the co-existence of three stable steady multi-cell states, a temporal periodic oscillation state, the co-existence of periodic oscillation and chaotic oscillation, a chaotic temporal oscillation, a subharmonic-bifurcation-driven asymmetric oscillating state, and a stable steady 2-cell state at large buoyancy force of heating.

1. Introduction

We study the fully-developed bifurcation-driven multiplicity and dynamic responses of multiple solutions to finite random disturbances numerically by the finite-volume/Euler-Newton continuation and the direct transient computation for the mixed convection in ducts of square cross-section with the streamwise curvature, the spanwise rotation in either positive or negative direction, and the wall heating/cooling [Fig. 1 with (R, Z, φ) as the radial, spanwise and streamwise directions, respectively]. A positive rotation

gives rises to a Coriolis force in the cross plane (RZ-plane) directed along positive R-direction and vice versa. Such flows and transport phenomena find their application in sedimentation field-flow fractionation, aerosol centrifuges, rotating power machinery, rotating heat exchangers, centrifugal material processing and material quality control, medical and chromatographic devices, etc.

Early works on the rotating curved duct flows were constrained to two simplified limiting cases with strong or weak rotations. Ludwieg (1951) developed a solution based on a momentum integral method for the isothermal flow in a square duct with a strong spanwise rotation. Miyazaki (1971, 1973) examined the mixed convection in a curved circular/rectangular duct with spanwise rotation and wall heating by a finite difference method. Because of the convergence difficulties with the iterative method used, Miyazaki's work was constrained to the case of weak curvature, rotation and heating rate. As well, all the works employ a steady model for the fully developed laminar flow with a positive rotation of the duct. Since the solution is only for the asymptotic cases, the secondary flow revealed by these early works consists of only one pair of counter-rotating vortices in the cross-plane. The interaction of the secondary flow with the pressure-driven streamwise flow shifts the location of the maximum streamwise velocity away from the center of the duct and in the direction of the secondary velocity in the middle of the duct.

More comprehensive studies have been made in recent years by Wang & Cheng (1996a) and Daskopoulos & Lenhoff(1990) for a circular tube, Matsson & Alfredsson (1990, 1994) and Guo & Finlay (1991) for a high-aspect-ratio rectangular duct, and Wang & Cheng (1995, 1996b, 1997, 2001), Wang (1997a, b, 1999), Selmi *et al.*(1994) and Selmi & Nandakumar (1999) for the square and rectangular ducts with a low-aspect-ratio. All the works are for the steady fully developed flows. Wang & Cheng (1996a) developed an analytical solution for rotating curved flow with effect of heating or cooling which allows to analyze the solution structure. Detailed flow structures and heat transfer characteristics were examined numerically by Wang & Cheng (1996b) and Wang (1997a, b, 1999). The rotating curved flows were visualized using smoke injection method by Wang & Cheng (1995, 1997, 2001). Daskopoulos & Lenhoff (1990) made the first bifurcation study numerically under the small curvature and the symmetry condition imposed along the tube horizontal central plane. Matsson & Alfredsson (1990) presented the first and comprehensive linear stability analysis. Matsson & Alfredsson (1994) reported an experimental study, by hot-wire measurements and smoke visualization, of the effect of rotation on both primary and secondary instabilities. Using a linear stability theory and spectral method, Guo & Finlay (1991) examined the stability of streamwise oriented vortices

to 2D, spanwise-periodic disturbances (Eckhaus stability). Detailed bifurcation structure and linear stability of solutions was determined numerically by Selmi *et al.* (1994) and Selmi & Nandakumar (1999) without imposing the symmetric boundary conditions.

It is the *relative* motion between bodies that determines the performances such as friction and heat transfer characteristics. The duct rotation introduces both centrifugal and Coriolis forces in the momentum equation describing the *relative* motion of fluids with respect to the duct. For isothermal flows of a constant property fluid, the Coriolis force tends to produce vorticity while the centrifugal force is purely hydrostatic, analogous to the Earth's gravitational field (Wang 2001). When a temperature-induced variation of fluid density occurs for non-isothermal flows, both Coriolis and centrifugal-type buoyancy forces could contribute to the generation of the vorticity (Wang 2001). These two effects of rotation either enhance or counteract each other in a nonlinear manner depending on the direction of duct rotation, the direction of wall heat flux and the flow domain. As well, the buoyancy force is proportional to the square of the rotation speed while the Coriolis force increases proportionally with the rotation speed itself (Wang 1997b). Therefore, the effect of system rotation is more subtle and complicated and yields new, richer features of flow and heat transfer in general, the bifurcation and stability in particular, for non-isothermal flows. While some of such new features are revealed by our recent analytical and numerical works (Wang 1997a, b, 1999, Wang & Cheng 1996a, b), there is no known study on the bifurcation and stability of mixed convection in rotating curved ducts.

The present work is a relatively comprehensive study on the bifurcation structure and stability of multiple solutions for the laminar mixed convection in a rotating curved duct of square cross-section (Fig. 1). The governing differential equations in primitive variables are solved for detailed bifurcation structure by a finite-volume/Euler-Newton continuation method with the help of the bifurcation test function, the branch switching technique and the parameterization of arc-length or local variable. Transient calculation is made to examine in detail the response of every solution family to finite random disturbances. The power spectra are constructed by the Fourier transformation of temporal oscillation solutions to confirm the chaotic flow. We restrict ourself to the hydrodynamically and thermally fully-developed region and two-dimensional disturbances. So far, a detailed 3D numerical computation of flow bifurcation and stability is still too costly to conduct. A 2D model is still useful for a fundamental understanding of rotating curved duct flows. However, our assumption of fully developed flow limits our analysis to the one preserving the streamwise symmetry. There may be further bifurcation to flows that breaks this symmetry and that

cannot be found in the present work.

2. Governing Parameters and Numerical Algorithm

Consideration is given to a hydrodynamically and thermally fully developed laminar flow of viscous fluid in a square duct with the streamwise curvature, the spanwise rotation, and the wall heating or cooling at a constant heat flux (Fig.1). The geometry is toroidal and hence finite pitch effect is not considered. The rotation can be positive or negative at a constant angular velocity. The duct is streamwisely and peripherally uniformly heated or cooled with a uniform peripheral temperature. The properties of the fluid, with the exception of density, are taken to be constant. The usual Boussinesq approximation is used to deal with the density variation. The gravitational force is negligible compared with the centrifugal and Coriolis forces.

Consider a non-inertial toroidal coordinate system (R, Z, ϕ) fixed to the duct rotating with a constant angular velocity about the $O'Z'$ axis, as shown in Fig. 1. We may obtain the governing differential equations, in the form of primitive variables, governing fully-developed mixed convection based on conservation laws of mass, momentum and energy. The boundary conditions are non-slip and impermeable, streamwise uniform wall heat flux and peripherally uniform wall temperature at any streamwise position. The proper scaling quantities for non-dimensionalization are chosen based on our previous experience (Wang & Cheng 1996b). The formulation of the problem is on full flow domain without imposing symmetric boundary conditions to perform a thorough numerical simulation. The readers are referred to Wang & Cheng (1996b) for the details of mathematical formulation of the problem.

The dimensionless governing equations contain five dimensionless governing parameters: one geometrical parameter σ (the curvature ratio defined by a/R_c , the ratio of duct width/height a over the radius of the curvature R_c, representing the degree of curvature), one thermophysical parameter Pr (the Prandtl number, representing the ratio of momentum diffusion rate to that of the thermal diffusion), and three dynamical parameters Dk, $L1$ and $L2$ defined in Wang & Cheng (1996b). The pseudo Dean number Dk is the ratio of the square root of the product of inertial and centrifugal forces to the viscous force and characterizes the effect of inertial and centrifugal forces. $L1$ represents the ratio of the Coriolis force over the centrifugal force, characterizing the relative strength of Coriolis force over the centrifugal force. $L2$ is the ratio of the buoyancy force over the centrifugal force and represents the relative strength of the buoyancy force. A positive (negative) value of $L1$ is for the positive (negative) rota-

tion. A positive (negative) value of $L2$ indicates the wall heating (cooling). In the present work, we set $\sigma = 0.2$ (typically used in cooling systems of rotor drums and conductors of electrical generators) and $Pr = 0.7$ (a typical value for air) to study the effects of three dynamical parameters on the multiplicity and stability. While results regarding the effects of Dk and $L1$ are also available, we focus on the effects of $L2$ at $Dk = 300$ and $L1 = 28$ in the present paper due to limited space.

The governing differential equations are discretized by the finite volume method to obtain discretization equations. The discretization equations are solved for parameter-dependence of velocity, pressure and temperature fields by the Euler-Newton continuation method with the solution branches parameterized by $L2$, the arc-length or the local variable. The starting points of our continuation algorithms are the three solutions at $Dk = 300$, $L1 = 28$ and $L2 = 0$ from our study of the effects of D_k and $L1$. The bifurcation points are detected by the test function developed by Seydel (1994). The branch switching is made by a scheme approximating the difference between branches proposed by Seydel (1994). The dynamic responses of multiple solutions to the 2D finite random disturbances are examined by the direct transient computation. The readers are referred to Wang & Yang (2001) and Yang & Wang (2000) for the numerical details and the check of grid-dependence and accuracy. The computations are carried out on the Super Computer SP2 of The University of Hong Kong.

3. Results and Discussion

3.1. SOLUTION STRUCTURE

The bifurcation structure is shown in Fig.2 for $L2$ values from -20 up to 70 at $\sigma = 0.02$, $Pr = 0.7$, $Dk = 300$ and $L1 = 28$. In Fig.2, the radial velocity component u at $r = 0.9$ and $z = 0.14$ (where the flow is sensitively dependent on $L2$; $r = R/a$ and $z = Z/a$) is used as the state variable, enabling the most clear visualization of all solution branches. Three solution branches, labeled by $AS1$, $AS2$ and $S3$ respectively, are found. Here, S stands for symmetric solutions with respect to the horizontal central plane $z = 0$, and AS indicates that the branch has both symmetric and asymmetric solutions.

Branch $AS1$ has sixty-nine limit points labeled by $AS1^1$ to $AS1^{69}$, eleven bifurcation points connecting its sub-branches denoted by $AS1^{AS1-1}$ to $AS1^{AS1-11}$, two bifurcation points connecting itself to $AS2$ labeled by $AS1^{AS2-1}$ and $AS1^{AS2-1}$, and one bifurcation point connecting itself to $S3$ denoted by $AS1^{S3}$. Branch $AS2$ has four limit points labeled by $AS2^1$ to $AS2^4$. Branch $S3$ is a symmetric solution branch and has two limit points $S3^1$ and $S3^2$. The location of fourteen bifurcation points and seventy-five

limit points is available in Yang (2001). To visualize the details of branch connectivity and some limit/bifurcation points, the locally enlarged state diagrams are also shown in Fig. 2. As Fig. 2 is only 1D projection of 12400 dimensional solution branches, all intersecting points except fourteen bifurcation points should not be interpreted as connection points of branches.

For a large $\mid L2 \mid$ value ($L2 \leq -14.5$ or $L2 \geq 63.1$), the buoyancy force dominant the mixed convection. There is unique flow and temperature field for a specified value of $L2$ in these two ranges. Figure 3 illustrates the secondary flow patterns, the streamwise velocity profiles and temperature profiles at $L2 = -17$ and $L2 = 65$, respectively. In the figure, the stream function, streamwise velocity and temperature are normalized by their corresponding maximum absolute values $\mid \psi \mid_{max}$, w_{max} and t_{max}. A vortex with a positive (negative) value of the secondary flow stream function indicates a counter-clockwise (clockwise) circulation. The readers are referred to Wang & Cheng (1996b) for a detailed discussion of the flow structures shown in Fig.3 in general, their relations with physical mechanisms and driving forces and their effects on the flow resistance and heat transfer in particular.

For a $L2$ value in $-14.5 < L2 < 63.1$, however, we can have multiple solutions. Figure 4 shows typical secondary flow patterns of six solutions (thirty-nine solutions in total) at $L2 = -11.7$. It is observed that the nonlinear competition of driven forces leads to not only a rich solution structure but also complicated flow structures. Therefore, the mixed convection in rotating curved ducts is much more complicated than that available in the literature.

3.2. STABILITY OF MULTIPLE SOLUTIONS

Recognizing that there is no study on dynamic responses of multiple solutions to finite random disturbances in the literature, a relatively comprehensive transient computation is made to examine the dynamic behavior and stability of typical steady solutions with respect to four sets of finite random disturbances with $d = 4\%, 10\%$, 15% and 40% respectively. Here, d is the maximum percentage of disturbing value over the steady value (Wang & Yang 2001).

Seven sub-ranges are identified with each having distinct dynamic responses to the finite random disturbances. The first ranges from $L2 = -20$ to $L2 = -14.5$, where the finite random disturbances lead all steady solutions at any fixed $L2$ to a steady symmetric multi-cell state on $AS1_a$ with the same $L2$. The second covers the range $-14.5 < Dk \leq -13.6$ where there is co-existence of three stable steady symmetric multi-cell states. In the third sub-range $-13.6 < L2 \leq 12.1$, all steady solutions evolve to

a temporal periodic solution. The fourth sub-range is from $L2 = -12.1$ to $L2 = -11.5$ where the solutions response to the finite random disturbances in the form of either periodic oscillation or chaotic oscillation. There is the co-existence of periodic and chaotic oscillations. In the fifth sub-range $-11.5 < L2 \leq -10.5$, all steady solutions evolve to a temporal chaotic solution. The next sub-range $-10.5 < L2 \leq -10.2$ serves as a transition between the chaotic oscillation and the stable steady 2-cell flow. The solutions response to the finite random disturbances in the form of subharmonic-bifurcation-driven asymmetric oscillation. In the last sub-range $L2 > -10.2$, the finite random disturbances lead all steady solutions at any fixed $L2$ to a stable steady symmetric 2-cell state on $AS1_s$ with the same $L2$. A detailed discussion of stability features can be found in Yang (2001).

4. Concluding Remarks

The governing differential equations from the conservation laws are discretized by the finite volume method to obtain discretization equations, a set of nonlinear algebraic equations. The discretization equations are solved for parameter-dependence of flow and temperature fields by the Euler-Newton continuation with the solution branches parameterized by $L2$, the arclength or the local variable. The bifurcation points are detected by the test function. The branch switching is made by a scheme approximating the difference between branches proposed. One symmetric and two symmetric/asymmetric solution branches are found with fourteen bifurcation and seventy-five limit points. Both solution and flow structures are much more richer than those available in the literature.

The dynamic responses of multiple solutions to the 2D finite random disturbances are examined by the direct transient computation. The finite random disturbances are found to lead the steady solutions to a stable steady multi-cell state in $-20 < Dk \leq -14.5$, the co-existence of three stable steady multi-cell states in $-14.5 < L2 \leq -13.6$, a temporal periodic oscillation in $-13.6 < Dk \leq -12.1$, the co-existence of periodic and chaotic oscillating states in $-12.1 < L2 \leq -11.5$, a chaotic oscillation in $-11.5 < L2 \leq -10.5$, a subharmonic-bifurcation-driven asymmetric oscillation in $-10.5 < L2 \leq -10.2$, and a stable steady 2-cell state in $-10.2 < L2 \leq 70$.

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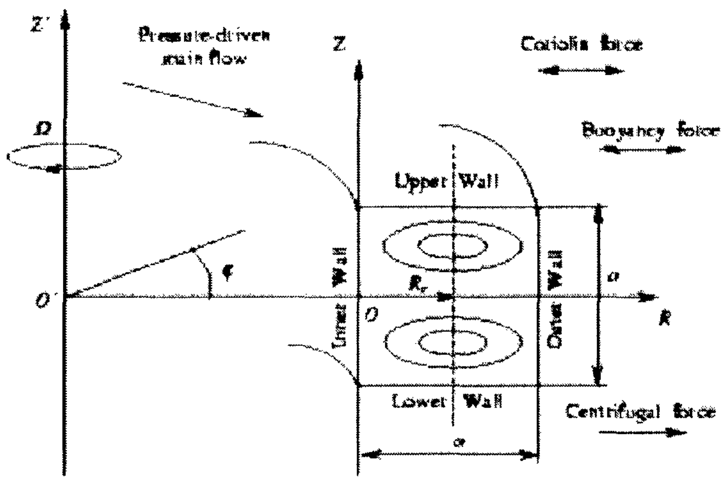
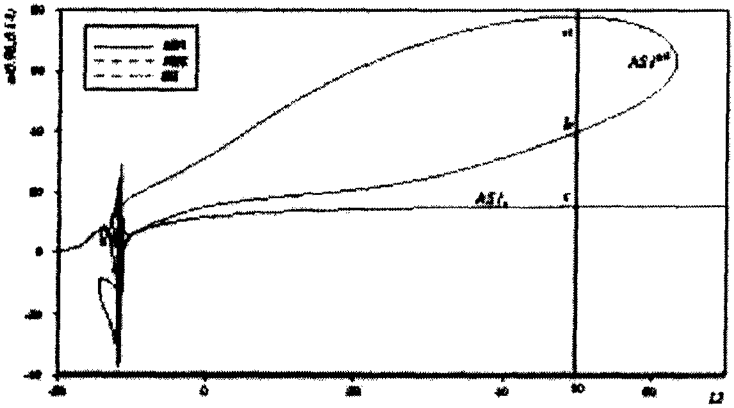
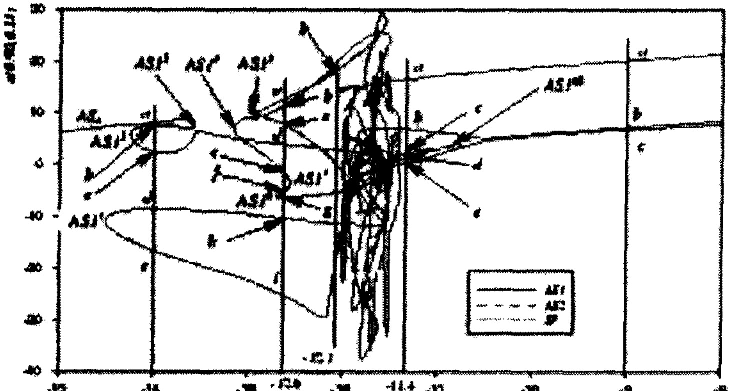


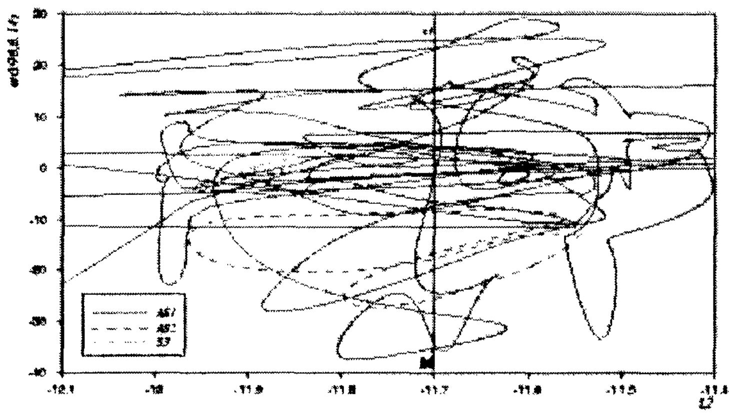
Figure 1. Physical problem and coordinate system



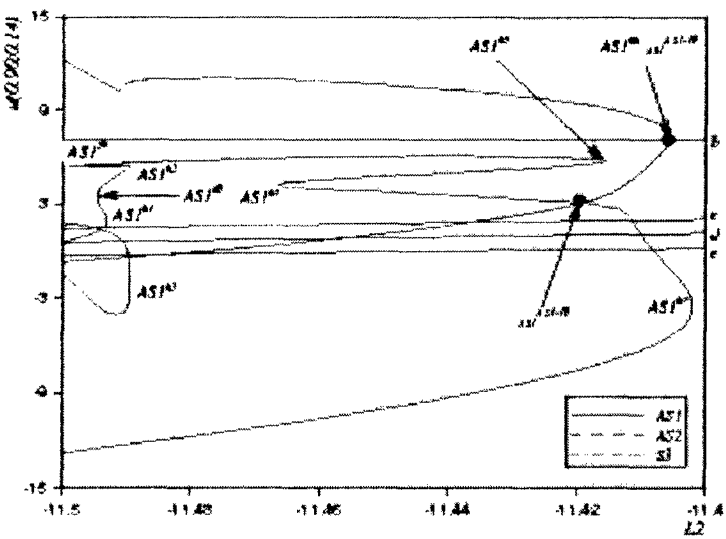
(a) $-20 < L2 < 70$



(b) $-15 < L2 < -8$

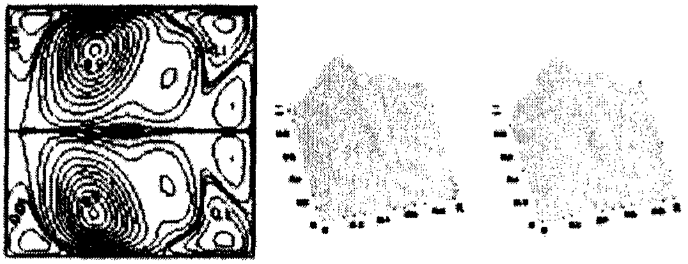


(c) $-12.1 < L2 < -11.4$

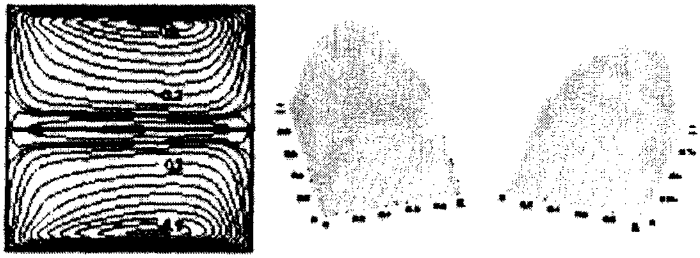


(d) $-11.5 < L2 < -11.4$

Figure 2. Solution branches and limit/bifurcation points ($\sigma = 0.02$, Pr=0.7, $Dk = 300$ and $L1 = 28$)



(a) $L2=-17.0$ on AS1 ($|\psi|_{max}=7.14$, $w_{max}=0.0606$, $t_{max}=127.3$)



(b) $L2=65$ on AS1 ($|\psi|_{max}=14.22$, $w_{max}=0.0137$, $t_{max}=48$)

Figure 3. Flow and temperature fields ($\sigma = 0.02$, Pr=0.7, $Dk = 300$ and $L1 = 28$; left: secondary flow, middle: streamwise velocity, right: temperature)

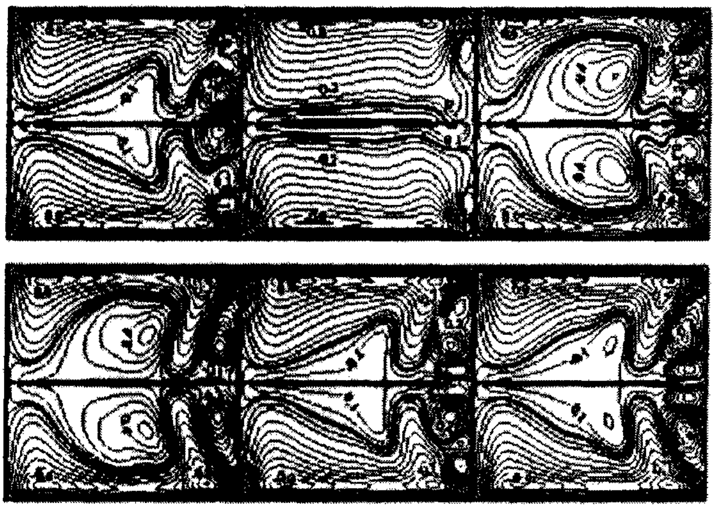


Figure 4. Typical secondary flow patterns of six solutions among thirty-nine solutions at $L2 = -11.7$ ($\sigma = 0.02$, Pr=0.7, $Dk = 300$ and $L1 = 28$)